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AVAILABILITY OF
MAINTAINED SYSTEMS

THESIS

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Preface

This thesis is basically an extensive survey done in the area of Availability as an important measure of system effectiveness. Availability appears to be a more appropriate measure than reliability for measuring the effectiveness of maintained systems because it includes reliability as well as maintainability.

I would like to thank my thesis advisor, Professor A. H. Moore, for his most valuable advice and guidance during this study. I would also like to thank Dr. Joseph Cain, my reader, for his help during the study. Also, I am grateful to Mrs. Phyllis Reynolds for her help in typing this thesis. I am also grateful to Mrs. Linda Stoddart of the Air Force Institute of Technology Library, for her help in obtaining several references.

Finally, a wish to recogize the wonderful effort of my wife, Eglal, who encouraged me to strive, to search, to study and, ultimately, to succeed.

— Ahmed A. El Shanawani

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Abstract

Availability appears to be a more appropriate measure than reliability for measuring the effectiveness of maintained systems because it includes reliability as well as maintainability. This thesis is a survey and a systematic classification of the literature relevant to availability. Emphasis in this thesis is centered on a variety of topics related to availability. The topics discussed are: the definition and concepts of the availability, the probability density functions of failure times and of repair times, system configurations; and the various approaches employed to obtain the availability models; effect of preventive maintenance policies on availability; availability parameters in the model; and system optimization.

AVAILABILITY OF MAINTAINED SYSTEMS

CHAPTER I

INTRODUCTION

Increasing complexity of modern-day equipment, both in the military and commercial areas, has brought with it new engineering problems involving high performance, reliability and maintainability. Reliability has long been considered as a measure of system effectiveness. However, it has proved to be an incomplete measure of effectiveness because it does not consider maintainability, another measure of system performance. With increasing complexity and the resulting high operational and maintenance costs, greater emphasis has been placed on reducing system maintenance while improving reliability. In this regard, availability, which is a combined measure of reliability and maintainability, has received wide usage as a measure of maintained systems effectiveness.

This thesis is a survey and a systematic classification of the literature relevant to availability. Emphasis in this thesis is centered on a variety of topics related to availability. In Chapter II, basic concepts include definition and concepts of availability, failure and repair times distributions, and system configuration.

In Chapter III, the different approaches used in obtaining availability models are discussed. In Chapter IV, many availability models using the Markovian approach are discussed. In Chapter V, the effect of preventive maintenance policies on availability is explained and classification of the availability parameters used in the model and system optimization is presented.

CHAPTER II

SURVEY ON BASIC ELEMENTS OF AVAILABILITY

In describing the availability of a given system it is necessary to specify three things:

1. The component failure process,
2. The repair or maintenance process, and
3. System configuration.

In this chapter, these three characteristics will be studied; but before exploring these characteristics, we would like to discuss the various definitions of availability.

Definition and Concepts of Availability

There are two classifications for availability.

Classification 1

In this classification the definition depends on the time interval; availability is classified into three categories (Figure 2.1): (1) instantaneous availability, (2) average uptime, and (3) steady-state availability [135].

1. Instantaneous availability, $[A(t)]$, is defined as the probability that the system is operational at any random time, t.

2. Average uptime availability, $[A(T)]$, is the proportion of time in a specified interval (0, T) that the system is

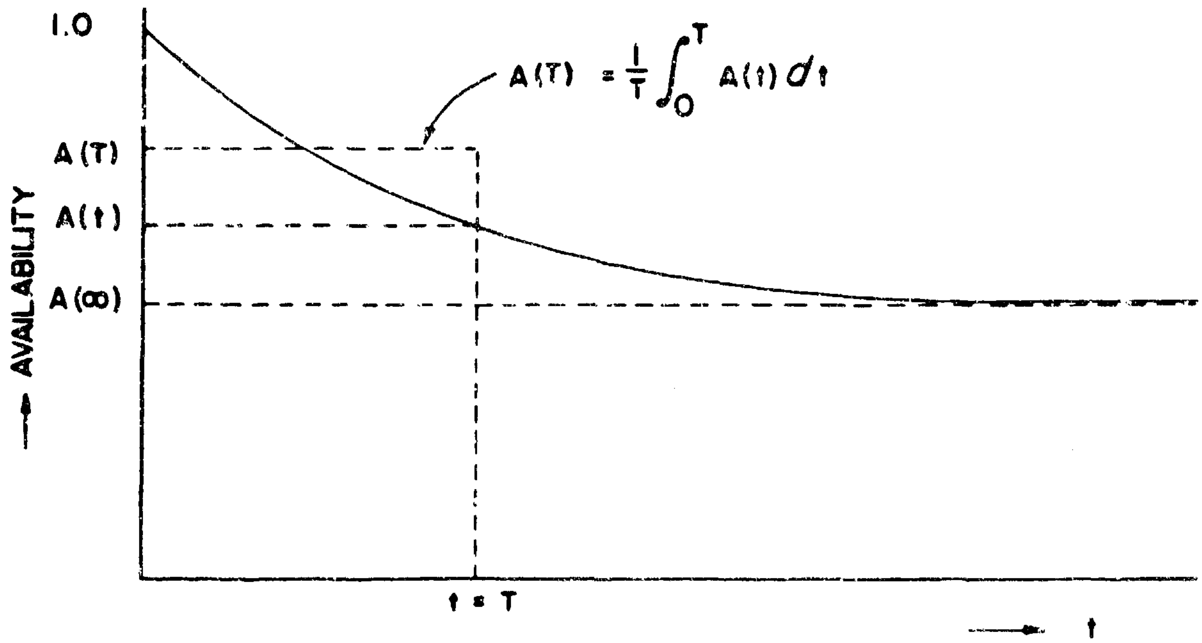

Fig. 2.1. Graph Showing Instantaneous, Average Uptime, and Steady State Availability

available for use and is expressed as:

$$A(T) = \frac{1}{T}\int_0^T A(t)\,dt \qquad (2.1)$$

3. Steady stage availability, $A(\infty)$, is the uptime availability when $T \to \infty$ and is given by:

$$A(\infty) = \lim_{t\to\infty} A(T) \qquad (2.2)$$

The representation of availability which is appropriate depends upon the system mission and its conditions of use. The steady-state availability may be the satisfactory measure for systems which are to be operated continuously. The average uptime may be the most satisfactory measure for systems whose usage is defined by a duty cycle. For systems which are required to perform a function at any random time, the instantaneous availability may be the most satisfactory measure.

Classification 2

In this classification the definition depends on the type of downtime. Availability is classified also into three categories: (1) inherent availability, (2) achieved availability, and (3) operational availability (Figure 2.2).

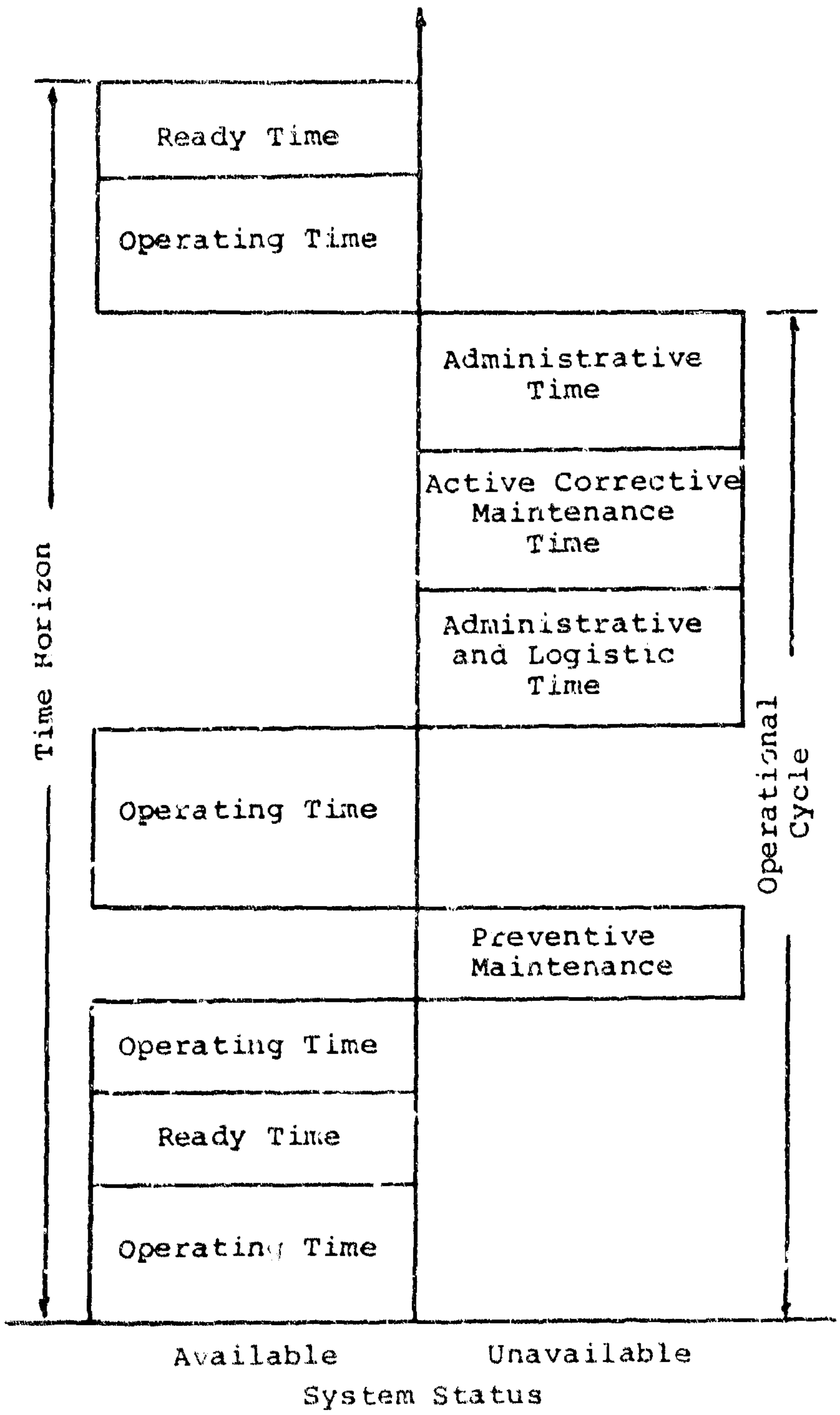

Fig. 22. A Graphical Description of System Status Over Time Horizon

In this category, the form used to describe system availability is that of an expected value function which assumes steady-stage conditions.

1. Inherent availability, A_i, is defined as the probability that a system, when used under stated conditions, without considering any scheduling or preventive action, in an ideal support environment, will operate satisfactorily at a given point in time. It excludes ready time, preventive-maintenance downtime, logistic time, and waiting or administrative downtime. It may be expressed as:

$$A_i = \frac{MTBF}{MTBF + MTTR} \qquad (2.3)$$

where:

MTBF = mean time between failure, and

MTTR = mean time to repair.

2. Achieved availability, A_a, is defined as the probability that a system, when used under stated conditions in an ideal support environment (i.e., available tools, spares, manpower, etc.), will operate satisfactorily at a given point in time. It excludes logistic time and waiting or administrative downtime. It includes active preventive and corrective maintenance downtime. It can be expressed as:

$$A_a = \frac{MTBM}{MTBM + M} \qquad (2.4)$$

where:

MTBM = mean time between maintenance, and

M = Mean maintenance time resulting from both corrective and preventive maintenance actions.

3. Operational availability, A_o, is defined as the probability that a system, when used under stated conditions in an actual operational environment, will operate satisfactorily at a given point in time. It includes ready time, logistic time, and waiting or administrative downtime. It can be expressed as:

$$A_o = \frac{\text{MTBM + Ready Time}}{\text{(MTBM + Ready Time) + MDT}} \quad (2.5)$$

where:

Ready time = the time in which the system is ready but not in operation,

MDT = Maintenance downtime including logistic downtime and waiting or administrative time, and

MDT = M + delay time.

Operational availability appears to be a more realistic measure than the other two measures. However, because delay time is determined by administrative and supply factors which depend on the environment of the system, this definition will not be used.

The Failure Process Distributions

The failure times distributions describe the component failure process; i.e., the probability law governing failures. There are two ways of postulating a component failure distribution:

1. Physical reasoning theory. In this method, we depend on physical reasoning to assume a form of the failure distribution. This method is useful when there is little a priori information.

2. Using observed empirical evidence. In this method, attempts can be made to fit a failure density function to the available data.

Of course, a combination of these two methods is optimal if sufficient statistical data are available and insight into the failure distribution can be obtained by physical theory.

Many types of failure distributions have been used in the literature. Classification of references on availability according to various types of failure time distributions (exponential, Erland, Weibull, Gamma, Rayleigh, normal, log-normal, uniform, extreme value, and general) is given in Table 2.1.

The most frequently employed distribution is the negative exponential distribution. To justify the use of the exponential failure law, much experimental and operational data have been collected. One of the earliest

TABLE 2.1

CLASSIFICATION OF REFERENCES ON AVAILABILITY WITH REGARD TO FAILURE TIME DISTRIBUTIONS

Name of Distribution	References
Exponential	1-4, 7-10, 14, 16, 18, 20-25, 28, 29, 35, 39, 41-43, 47, 48, 50, 53-57, 59, 60, 63, 65-70, 74-77, 83, 86-88, 90, 93, 94, 96, 97, 103, 106, 109, 112, 113-122, 126-128, 130, 137, 139, 140, 143-145, 150, 152, 154-158, 164, 165, 167, 168-173, 175-179, 192, 193
Erlang	41, 91, 104, 151, 157, 165
Weibull	10, 16, 41, 88, 112, 113, 157, 165, 179, 193, 196
Gamma	10, 16, 41, 88, 112, 113, 157, 165, 179, 182
Rayleigh	112, 116, 165
Normal	10, 16, 21, 41, 56, 112, 113, 117, 165, 179, 182
Log-Normal	10, 14, 16, 40, 58, 113
Uniform	27, 116, 165
Extreme Value	10, 113
General (Arbitrary)	19, 20, 30, 47, 51, 66-68, 105, 110-112, 126, 131, 133-136, 142, 144, 162, 166, 190

reports of a statistical nature was made by Davis [49], and subsequent studies by Carhart [37] and Boodman [22] indicate that this distribution adequately fits failure experience. Cox and Smith [46] demonstrate that the equipment generally will exhibit the exponential failure pattern provided that the components are replaced as they fail, even though certain components within the equipment may not exhibit it.

This distribution seems to apply to all electronic equipment. The rationale behind this is that the electronic components do not fail from wearout or fatigue, but from being overstressed; and these overstressed conditions are purely randomly distributed. In addition, all military standards and 90 percent of the military reliability calculations are based on random failures [112]. The most attractive feature in using the exponential distribution is that it enables one to deal with a constant failure rate. Hence, it provides an advantage from a mathematical tractability point of view even though it is not always justified.

Bocchi [21] demonstrated the suitability of using the exponential failure distribution for mechanical reliability prediction. The rationale for that is during the useful life period when failures are due to poor quality and wearout is low, failure rates should tend to be somewhat constant. The main contributor to the failure rate is when random high stress levels exceed the strength of the components. Other components which also justify the use of exponential failure distributions are tube puncture, capicitor breakdown, fuse blowout, many aircraft and missile parts, airborne radars and fire control systems. References that justify the use of the exponential failure distribution are References 22, 37, 46, 49, and 196.

After the exponential distribution, the Weibull distribution is probably the most widely used distribution. The hazard function of the Weibull given by

$$h(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1}, \qquad t \geq 0 \tag{2.6}$$

will decrease in time if $\beta < 1$, will increase if $\beta > 1$, or will be constant if $\beta = 1$ which is the exponential case. The Weibull distribution has been used to describe fatigue failure, vacuum tube failure, and ball bearing failure. It is the most popular parametric family of failure distributions.

The Raleigh distribution is a single parameter density which holds for a component with a linearly increasing failure rate (λt).

The rectangular or uniform distribution may well be employed if every component has the same failure rate or each item takes equally as long to repair.

The Erlang distribution is used to describe both the failure and repair times. Kodama [104] used the

Erlang as a failure distribution. Since the Erlang distributions are a special case of the incomplete gamma distributions (shape parameter is an integer), they will fit many an perhaps most of the distributions encountered in practice, and mathematical treatment will be easy.

The normal distribution describes wearout failures. By wearout failures we mean those cases in which no overt or abrupt failure has occurred but the item has more or less gradually reached the failed state through the deterioration or depletion of some quantity, structure, or function necessary for useful operation. In this type of failure it is noticed that the component's death tends to cluster around a mean life time, $\bar{t}$; half the failures occurring before and half afterward. There are few very early or very late failures, the failure rate being low initially and reaching a maximum at the mean lifetime. The hazard is very low initially, and rises rapidly after $\bar{t}$. This familiar pattern of failure can be described by the normal distribution [37] in which the failure rate as a function of operating time, t, is given by:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \tag{2.7}$$

The normal distribution failure pattern applies to systems which exhibit small variation in failure resistance among the individuals within a population and which are subject to small variations in environmental severity. Further, the failure resistance of the mechanism deteriorates with time and operational procedure requires that each item be used until ultimate failure. Davis [49] states that the normal distribution characterizes the failure of dry cells and light bulbs. Bell [16] mentioned also that vacuum tubes used in commercial and military electronic equipment follows the normal failure rate besides significant fraction of the commercial aircraft parts.

Many life length distributions occurring in practical applications are obviously not normal because they are markedly skewed whereas the normal distribution is symmetric. The gamma family of distributions is skewed and therefore may seem more natural than the normal family in these cases.

The gamma density function is described by:

$$f(t) = \frac{\lambda(\lambda t)^{\alpha-1} e^{-\lambda t}}{\Gamma(\alpha)} \qquad \lambda, \alpha > 0, \; t \geq 0 \tag{2.8}$$

The gamma has increasing failure rate for $\alpha > 1$ and, in this case, the failure rate is bounded above by λ; for $\alpha < 1$, the failure rate is decreasing.

The log-normal density is defined as:

$$f(t) = \frac{1}{t\sigma\sqrt{2\pi}} \exp[-\frac{1}{2\sigma^2}(\log t - \mu)^2] \qquad \begin{array}{c} -\infty < \mu < \infty \\ \sigma > 0 \\ t \geq 0 \end{array} \tag{2.9}$$

This is a skew distribution in which both long and short downtimes occur more frequently than would be the case in data with the same value fitted to an exponential distribution. The failure rate of the log-normal distribution increases at first and then eventually decreases to zero. For this reason, the log-normal has found disfavor as a failure distribution. It has been proposed as a reasonable family of distributions for describing the length of time to repair a piece of equipment, however, and there is some empirical evidence for this assertion [10].

Many authors including Coppola [45] and Howard [92], indicate that downtimes are generally well fitted by a log-normal distribution. Shelley [163] pointed out the use of log-normal for cargo aircraft perfectly fits the data, especially at the upper percentile points. Recent reliabilities studies on various potential communication systems indicates that many semiconductor devices have lifetime distributions well represented by the log-normal [40].

On the basis of actual observation of time to failure it is difficult to distinguish among the various nonsymmetrical probability functions. Thus, the differences among the gamma, Weibull, and log-normal distribution functions become significant only in the tails of the distribution but actual observations are sparse in the tails because of limited sample sizes.

The Repair Process Distributions

Table 2.2 shows the classification of references on availability with regard to a variety of repair time distributions: exponential, Erlang, Weibull, Gamma, Rayleigh, normal, log-normal, uniform, and general.

The exponential distribution is used as a theoretical distribution for the repair time because of its analytical properties and computational purposes [188]. Rohn [154] maintains that the essential characteristic of repair times of complex electronic equipment is stated as a high frequency of short repair times and a few long repair times; thus, this type of behavior suggests representation by an exponential distribution.

As mentioned before, the log-normal distribution is quite popular for the distribution of repair times. In many situations, repair times are best described by the log-normal distribution, and many authors [45, 92, 163, 179, 187] justify the use of the distribution. Studies on airborne radar equipment and ground equipment for surface-to-air missile systems have indicated observed repair time distributions that best fit the log-normal distribution [77, 162].

TABLE 2.2

CLASSIFICATION OF REFERENCES ON AVAILABILITY WITH REGARD TO REPAIR TIME DISTRIBUTIONS

Name of Distribution	References
Exponential	1-4, 7, 10, 18, 23, 24, 25, 35, 39, 43, 50, 53-56, 59, 63, 68-70, 74-75, 86-88, 90, 93, 94, 103, 107, 112, 114, 116, 118, 120, 122, 127, 137, 139, 140, 143, 154, 156-158, 165, 172, 173, 175, 188, 192, 193
Erlang	69, 122, 126, 144
Weibull	29, 112, 193
Gamma	24, 29, 116, 140, 144, 146, 157
Raleigh	112, 116
Normal	14, 20, 47, 56, 112
Log-Normal	10, 20, 29, 47, 56, 60, 83, 88, 102, 179
Uniform	116, 122
General (Arbitrary)	10, 19, 28, 30, 42, 43, 48, 51, 65, 74, 76, 96, 97, 104, 105, 106, 109, 110-112, 119, 121, 126, 130, 131, 133-136, 142, 144, 145, 150, 151, 162, 164, 166, 167, 168, 171, 190

System Configurations

Classifications of references on system configuration are shown in Table 2.3. The logical approach in the availability analysis is to decompose the system under consideration into functional entities composed of components or subsystems. This subdivision generates a block-diagram and describes the system operation. To fit this logical structure, models are formulated. In this way, the block-diagram of the type of the system configurations describes how the components are functionally connected and the rules of operation.

The simplest structure in availability analysis is the single configuration in which only one component comprises a system.

The series configuration is the next simplest and most common structure. In this configuration the functional operation of the system depends on the operation of all system components. The redundant configuration can be divided into two main categories--the parallel redundant configuration and the standby redundant configuration. In the parallel redundant configuration the system operates if any one of the components operate. This configuration is often called the full redundant configuration. On the other hand, if the system operation requires more than one component to operate, this configuration is called the partial redundant configuration. In the parallel system all

TABLE 2.3

CLASSIFICATION OF REFERENCES ON AVAILABILITY
WITH REGARD TO SYSTEM CONFIGURATIONS

System Configuration	References
Single	6, 7, 10, 14, 25, 28, 35, 39, 53, 75, 105, 114, 116, 156, 157, 165, 179, 182, 193
Series	10, 14, 23, 53, 78, 90, 96, 119, 126, 130, 142, 143, 160, 164, 165, 173, 174, 179, 190
Redundant	
Parallel Redundant	2-5, 7, 10, 14, 24, 35, 39, 54, 59, 63, 68, 74, 75, 77, 85, 87, 88, 90, 94, 103, 104, 111, 118, 120-122, 139, 140, 143, 152-155, 157, 158, 165, 173, 179, 192, 193
Standby Redundant	4, 10, 13, 14, 19, 30, 39, 42, 43, 48, 55, 59, 65, 68, 77, 78, 88, 93, 104, 109, 121, 128, 132-137, 140, 142, 144, 152, 157, 165-167, 168, 171, 179
Perfect Switch	13, 19, 30, 39, 42, 43, 65, 78, 104, 109, 136-139, 166, 171
Imperfect Switch	48, 96, 137, 140, 142
Cold Standby	13, 30, 39, 59, 65, 76, 78, 79, 125, 128, 134, 136, 137, 160, 166
Warm Standby	19, 42, 43, 104, 144, 167, 170, 171
Series Parallel	10, 39, 54, 65, 75, 106, 110, 112, 143, 165, 175, 179
Complex	60, 90

the components are turned on at the beginning and operate until failure occurs. Using less reliable units in redundant configurations is one of the methods of coping with the problem of designing reliable systems. For nonmaintained systems, redundancy is best applied at the component level rather than at the system level. However, for systems whose components can be repaired as they fail, to have redundancy at the component level may not be the best policy. The reason is that if component redundancy is employed, repair may not be possible while the system is operating; whereas, a failure with system redundancy could be repaired.

In the standby redundant system the parallel components are not active at the same time. At the start of operation the switch connects the input to one component. Meanwhile, other components are left in standby with zero failure rate or a failure rate lower than the active components. The system in which standby components cannot fail is then referred to as cold standby. The system is called warm standby if only one component operates at a time, and the standby component has a lower failure rate than the active component, but not zero failure rate as in cold standby.

The standby configuration can be divided according to the type of switching to two types: (1) perfect switching, and (2) imperfect switching. If the switching device is assumed to be perfect, the standby system is better than the parallel system. The situation changes when the standby component ages and the switch is imperfect. Figure 2.3 represents the different types of system configurations.

Based on the configurations discussed above, the system configuration concept is further extended to include series parallel, parallel series, and complex. By complex configuration we mean a system which is not purely series, parallel, series parallel or parallel series.

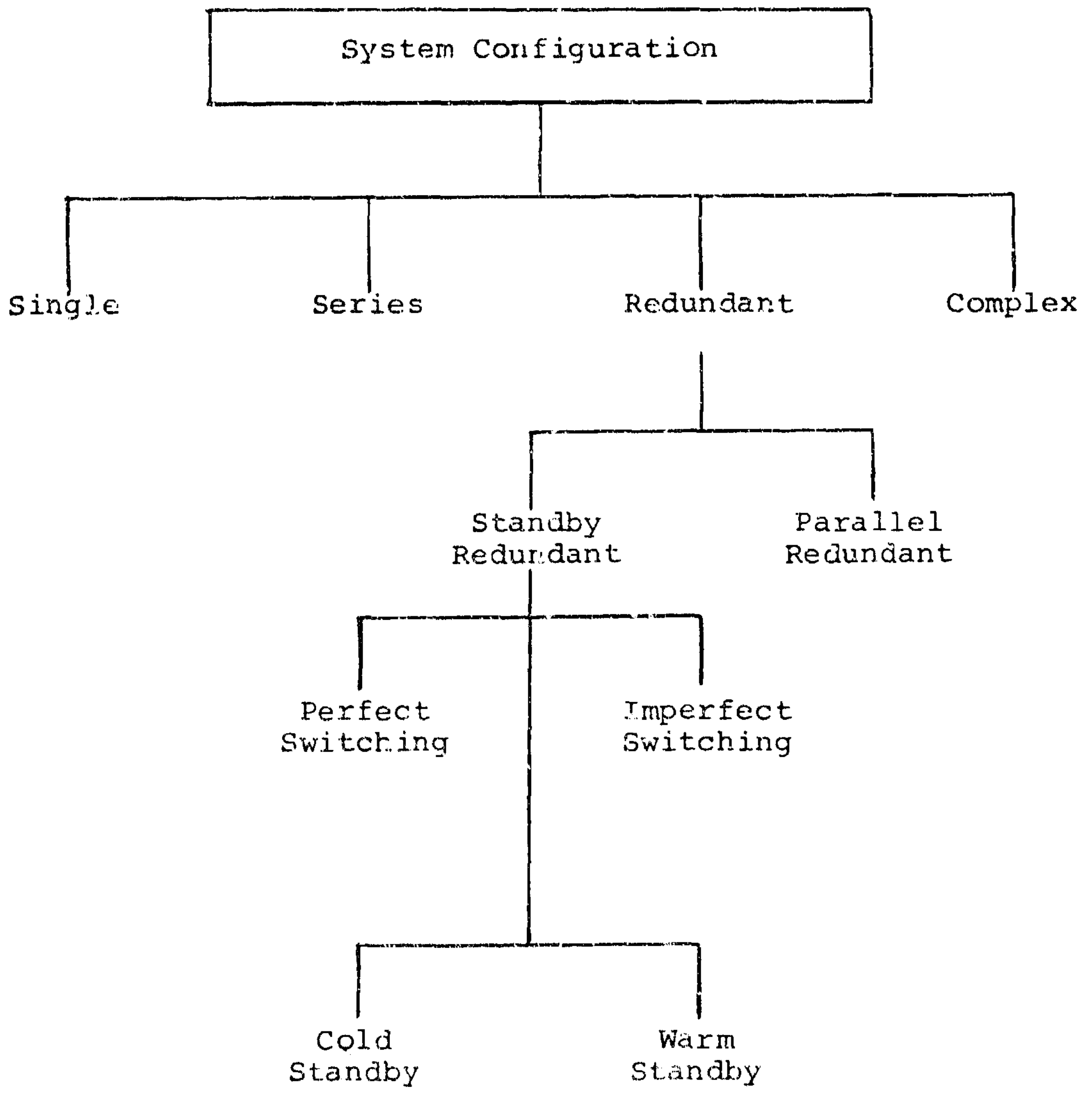

Fig. 2.3. Different Types of System Configurations

CHAPTER III

APPROACHES USED IN OBTAINING AVAILABILITY MODELS

Markovian

The Markovian approach in the formulation of the availability model has been frequently used assuming exponential distributions for failure times and repair times (see Table 3.1 for references). To obtain the availability model of a given system using this approach, Sandler [157] suggests that the following to be specified: (1) the component failure process, (2) the system configuration, (3) the repair policy, and (4) the state in which the system is defined to be failed (see Chapter IV for details).

For an illustration, let us consider a single component system with a constant failure rate, λ, and a constant repair rate, μ (exponential distribution). Since repair is possible, transitions can be made back and forth. Thus, two states can be designated: (1) State 0--the system is operating, and (2) State 1--the system has failed and is under repair.

Using conditional probabilities, the transition matrix can be constructed and the differei ial equations

TABLE 3.1

APPROACHES USED IN OBTAINING AVAILABILITY MODEL

Classification	References
Markovian	
Instantaneous Availability	10, 19, 25, 39, 63, 69, 72, 93, 107, 127, 132, 134-137, 153, 157, 165, 166, 178, 192
Average Uptime Availability	10, 39, 63, 69, 157
Steady-State Availability	2, 3, 5, 10, 24, 25, 39, 42, 50, 53-56, 59, 63, 69-74, 78, 79, 87, 90, 93, 94, 103, 109, 111, 114, 120, 134-137, 139, 140, 156, 157, 160, 165, 167, 171, 175
Ratio of Uptime to Total Time	1, 4, 14, 20, 23, 35, 47, 51, 60, 65, 68, 75, 83, 89, 92, 96, 100, 110-112, 116, 119, 120, 130, 131, 143, 158, 162, 172-174, 188, 190, 193
$\frac{\text{MTBF}}{\text{MTBF+MTTR}}$	4, 20, 23, 51, 65, 75, 89, 92, 96, 110, 119, 120, 126, 143, 158, 172, 173, 190, 193
$\frac{\text{MTBM}}{\text{MTBM+M}}$	20, 51, 112
$\frac{\text{Uptime}}{\text{Uptime+Downtime}}$	20, 51, 60, 111, 116, 131, 164, 174
Integral Theory	68
Monte Carlo Simulation	60, 123
Single-Cycle Availability	116, 131
Multiple-Cycle Availability	96
Confidence Interval of Availability	25, 29, 131, 172-174
Bayesian Approach	24, 25, 73, 173, 174

describing the stochastic behavior of the system can be formed.

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_1(t) \tag{3.1}$$

$$\frac{dP_1(t)}{dt} = \lambda P_0(t) - \mu P_1(t) \tag{3.2}$$

where:

$P_i(t)$ denotes the probability of the system being in state i at time t.

If the system is in operation at time t = 0, the initial conditions are $P_0(0) = 1$ and $P_1(0) = 0$. Transforming equations (3.1) and (3.2) into Laplace transforms under the above initial conditions, we have

$$(s+\lambda)P_0(s) - \mu P_1(s) = 1 \tag{3.3}$$

$$-\lambda P_0(s) + (s+\mu)P_1(s) = 0. \tag{3.4}$$

Now the instantaneous availability, A(t), is the inverse Laplace transform of $P_0(s)$; i.e., $A(t) = \mathcal{L}^{-1}\{P_0(s)\}$. Solving

$$A(t) = P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \tag{3.5}$$

the average uptime for some definite period of time (0, T) can be found by integrating A(t) over this time interval and dividing by the total time.

$$A(T) = \frac{1}{T}\int_0^T A(t)\,dt = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 T}\left[1-e^{-(\lambda+\mu)T}\right] \tag{3.6}$$

If we are interested in the long-range availability, we can let $T \to \infty$ and find the steady-state availability

$$A(\infty) = \frac{\mu}{\lambda+\mu} \tag{3.7}$$

Due to analytical and computational difficulty, not much work has been done when failure and repair times are other than exponential. For the analysis of the redundant system with exponential failure pdf and the general repair time distribution, Branson and Shah employ a semi-Markov process. Hall and others [88] analyze the redundant system when failure times and repair times follow combinations of the exponential, Weibull, and log-normal distributions. They illustrate the use of Fourier series for evaluating the inverse Laplace transformation. Although non-Markovian processes have not been studied as widely as Markovian processes, Sandler [157] shows that it is often possible to treat a stochastic process of the non-Markovian type by reducing it to a Markov process. This can be done by increasing the number of states, each being described by a constant transition rate. As an example, a single component system with an Erlang failure distribution and the cdf

$$F(t) = 1 - e^{-\lambda t} - \lambda t e^{-\lambda t} \tag{3.8}$$

and an exponential repair distribution with the cdf

$$G(t) = 1 - e^{-\mu t} \tag{3.9}$$

by assuming that the component goes through two exponential phases each of average length $1/\lambda$, the process can be reduced to a Markov process with three states:
(1) State 0--the system is operating in the first phase,
(2) State 1--the system is operating in the second phase,
and (3) State 2--the system has failed and is under repair.

This formulation leads to the transition matrix:

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-\lambda & \lambda & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & 0 & 1 \end{pmatrix} \end{array} \tag{3.10}$$

The solution of this matrix is simply

$$R(t) = P_0(t) + P_1(t) = e^{-\lambda t}(1+\lambda t) \tag{3.11}$$

Regulinski [153] used the Markovian approach to model the availability function for computer networks. Gates [72] presented an analytic technique for evaluating the availability of complex systems which are required to operate around the clock, but which are staffed with maintenance personnel periodically on a shift basis. He shows that such systems can be modeled as a periodically, time varying Markov process governed by a repeatable sequence of transition matrices.

Doyon [56] utilizes the steady-state availability concept to analyze a computer system consisting of a data processor and tape units. The purpose of the analysis is to solve for the MTTR of the redundant system. The author points out that defining the system states and formulating the appropriate system steady-state availability transition rate diagram is the step requiring the greatest degree of ingenuity and expertise. By contrast, subsequent steps to obtain a numerical solution for the system MTTR involves only routine mathematical manipulations.

The above approach is called the differential theory in reliability since the states of the system can be expressed in the form of a set of differential equations whose solution permits the evaluation of reliability and availability of the system. When failure and/or repair time are not exponentially distributed, the differential theory is not applicable; so the integral theory was introduced to overcome differential theory limitations.

Integral Theory of Reliability

The first paper on integral theory was published in 1973. In 1974 integral theory was used to evaluate the reliability of complex systems, such as telephone exchanges, whose repair time was not exponentially distributed [Galetto, 68]. In 1975 it was proved that integral and differential theories are equivalent as Markovian processes are studied. In the same year, integral theory was applied to state a general model for system cost-effectiveness, as failure and repair rates are assumed constant. In 1977 Galetto used the differential theory for obtaining the reliability and availability of different system configurations and drive formulas for MTTR (mean time to repair), mean uptime (MUT) and mean downtime (MDT) as a function in MTTR and then to derive steady state availability, $A(\infty)$:

$$A(\infty) = \frac{MUT}{MUT + MDT} \tag{3.12}$$

Galetto shows that the ratio $\frac{MTTF}{MTTF + MTTR}$ is a meaningless definition of availability, unless series systems are considered.

The integral theory of reliability overcomes the limitation of the differential theory especially for the mechanical systems since the failure rate for such systems is increasing as they age during operation.

Ratio of Uptime to Total Time

Another approach in the formulation of the availability model is the use of the definitions inherent, achieved, and operational availability. When only corrective maintenance is considered, the inherent availability which is a function of MTBF and MTTR is employed. In this case, MTBF is computed by:

$$MTBF = \int_0^{\infty} R(t)dt \tag{3.13}$$

where:

R(t) is the reliability function of the system.

MTTR is interpreted as synonymous with mean corrective maintenance time. When both corrective and preventive maintenance are considered, the achieved availability which is a function of MTBM and M is introduced where MTBM is the mean interval of all maintenance requirements, both corrective and preventive. M is the downtime resulting from both corrective and preventive maintenance. For example, when preventive maintenance is scheduled at time, T, it is expressed by

$$MTBM = \int_0^{T} R(s)ds \tag{3.14}$$

M is expressed as:

$$M = \frac{M_c f_c + M_p f_p}{f_c + f_p} \tag{3.15}$$

where:

M is the downtime resulting from both corrective and preventive maintenance,

M_c is the mean corrective maintenance time,

M_p is the mean preventive maintenance time,

f_c is the number of corrective maintenance actions, and

f_p is the number of preventive maintenance actions.

Operational availability is an appropriate measure if downtime includes logistics and administrative time as well as active maintenance downtime. For the classification of references, see Table 3.1.

Monte Carlo Simulation

Whenever the problem is extremely complex and/or experimentation is desirable but costly, Myers suggests the use of the Monte Carlo technique, and illustrates a few examples of this solution technique. Faragher and Watson [60], however, maintain that availability analysis of complex systems utilizing Monte Carlo simulation technique have revealed a lack of realism because they are inflexible with respect to configuration changes, thus making them unsuitable for optimization studies of availability through component redundancy. By incorporating engineering and mathematical analysis, they present a realistic methodology which involves an engineering description of the system, the formulation of the simulation model, and the computer and engineering analysis of the system.

Single-Cycle Availability

The definition of availability given by the fraction of the total desired operating time has been quite widely used as a main design criterion. However, there is no probabilistic guarantee that a specified availability value will ever be reached other than approximately in practice. Martz [116], therefore, provides a definition of single cycle availability that incorporates a probabilistic guarantee that the availability value will be reached in practice. Single-cycle availability is defined as the value, A_v, such that:

$$P(A \geq A_v) = v \qquad 0 \leq v \leq 1 \qquad (3.16)$$

By specifying v we have a probabilistic guarantee on the frequency of occurrence of the corresponding availability value.

For example, if we require a system availability $A_v = 0.99$ and v is chosen to be 0.90, in this case, we are 90 percent certain that our design value of 0.99 will be met in practice. To illustrate the use of this definition, Martz [116] presents a few examples with exponential, uniform, and Rayleigh distributions for failure and repair times, and shows that the median cycle availability $A_{0.05}$ is equivalent to the steady-state availability. Nakagawa and Goel [131] extend the definition for Martz for a finite interval. Their definition differs with

Martz's in that they take into consideration the interval of system operation.

Availability for Multiple Cycles and for a Finite Time

Kabak [96] discusses two types of availability: (1) availability for a given number of cycles, and (2) availability for a given length of time. His concept of availability is the proportion of time that system is up and is denoted by

$$\frac{t}{t+R}$$

where:

t = failure time which has a distribution $f(t)$, and

R = a constant repair time.

The availability for one cycle, $A(1)$, is defined in terms of expected value of $\frac{t}{t+R}$; that is,

$$A(1) = \int_0^\infty \frac{t}{t+R} f(t)dt \tag{3.17}$$

For i cycles, the total elapsed time is $T + iR$ where $T = \sum_{j=1}^{j=i} t_j$; i.e., T is the i-fold convolution of t.

The availability for i cycles, $A(i)$, is the expected value of $\frac{T}{T+iR}$ and is given by:

$$A(i) = \int_0^T \frac{T}{T+iR}\, g(T)\, dt \qquad (3.18)$$

if t has exponential distribution. T has an Erlang distribution with i degrees of freedom.

The finite time availability is determined by considering the number of times, n, that the system has suffered a failure in the interval (0, T) where T is given, and by combining the associated probability with the proportion of available time.

In the limit when $T \to \infty$ the finite time availability approaches the steady-state availability.

Confidence Interval of Availability

A point estimate of availability has usually been the only statistic calculated, although decisions about the true availability of the system should take uncertainty into account. Uncertanities in the value of MTBF and MTTR reflect an uncertainty in the value of the point availability

$$A(t) = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

Treating these uncertain parameters as random variables, the distribution of the point availability can be derived by combining the distributions of the failure and repair times. Hence, constructing estimates and confidence statements for the availability which are consistent with the equivalent statements on the failure time and repair time parameters.

Thompson [172] derives techniques for placing a lower confidence limit on system availability and for deciding if the true system availability differs significantly from a specified value when MTBF and MTTR are estimated from test data. Assuming times to failure and times to repair are stochastically independent random variables that follow exponential distributions with MTBF = θ and MTTR = ϕ respectively, $(1 - \alpha)$ lower confidence limit (LCL) for A is obtained by:

$$\mathrm{LCL} = \frac{\hat{\theta}}{\hat{\theta}+\hat{\phi}F_{1-\alpha}(2n,2n)} \tag{3.19}$$

where:

$\hat{\theta}$ and $\hat{\phi}$ are sample estimates of θ and ϕ respectively, and

n is the number of failure or repair actions.

In a similar manner, a two-sided confidence interval is derived and given by:

$$\mathrm{LCL} = \frac{\hat{\theta}}{\hat{\theta}+\hat{\phi}F_{1-\alpha/2}(2n,2n)} \tag{3.20}$$

$$\mathrm{UCL} = \frac{\hat{\theta}F_{1-\alpha/2}(2n,2n)}{\hat{\theta}F_{1-\alpha/2}(2n,2n)+\hat{\phi}} \tag{3.21}$$

Butterworth and Nikolaisen [29] are also concerned with the bounds on the availability function for the exponential failure distribution and for the general repair time distributions. They employ the gamma, log-normal, and Weibull distributions as repair time distributions. A bound on the error is also given. Some numerical examples are given to illustrate the practicality of the bounds presented.

Bayesian Approach

The Bayesian approach in the formulation of availability models has been employed in several references (See Table 3.1). Brender [25] carries out the statistical assessment of system availability within a Bayesian framework. He considers an availability model consists of an alternating sequence of independent exponentially distributed operational and repair intervals, with the failure time and repair time parameters described by distinct gamma distributions. This model is further extended in Reference 24, in which a more general prior distribution is considered for the parameters consisting of a linear combination of gamma distributions. Furthermore, a non-exponential distribution with uncertain scale and shape parameters is introduced. Gaver and Mazumdar [73] provide an analysis for a particular class of sampling plans, with the ultimate goal of estimating the long-run system availability. They combine mixed data using snap-shot data along with subsystem life and repair data for a simple subsystem.

Thompson and Springer [174] extend this result for a snap-shot data to systems of several subsystems. Here, snap-shot data merely reveals whether the system is up or down at the instant when the observation is made and applies only where the state of each subsystem is recorded on successive observations. A generalization of Reference 73 to systems of N subsystems can be seen in Reference 173, where data consists of samples of subsystem life and repair times.

Brender [25] develops a Bayes transformation which utilizes the failure and repair data to readily convert prior estimates and confidence statements on the availability into posterior distributions. Thompson and Springer [174] also carry out a Bayes analysis of system availability for an N component series system. They determine the posterior pdf of the availability through the derivation of the pdf of the product of N independent random variables using the Mellin integral transform. Confidence limits on the system availability are then obtained from the knowledge of the posterior pdf of the availability.

A numerical procedure for computing Bayes confidence intervals for the availability can be seen in

Reference 173. Here, both the series and parallel systems are considered.

A list of references on this topic is in Table 3.1.

CHAPTER IV

SOME AVAILABILITY MODELS USING THE MARKOVIAN APPROACH

Single-Equipment Systems

In this case we have only one unit which can have one of two states: (1) State 0--the system is operating, and (2) State 1--the system has failed and is under repair. Assuming that the failure rate is constant λ; i.e., the failure distribution is exponential and also the repair distribution is exponential with mean μ. Now since the conditional probability of failure in t, t+dt is λdt and the conditional probability of completing a repair in t, t+dt is μdt, we have the following transition matrix:

$$P = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 1-\lambda & \lambda \\ \mu & 1-\mu \end{pmatrix} \end{array} \qquad (4.1)$$

The system is depicted in Figure 4.1.

The differential equations describing the stochastic behavior of this system can be formed by considering the following:

The probability that the system is in State 0 at time t+dt is derived from the probability that it was in State 0 at time t and did not fail in t,t+dt, or that it

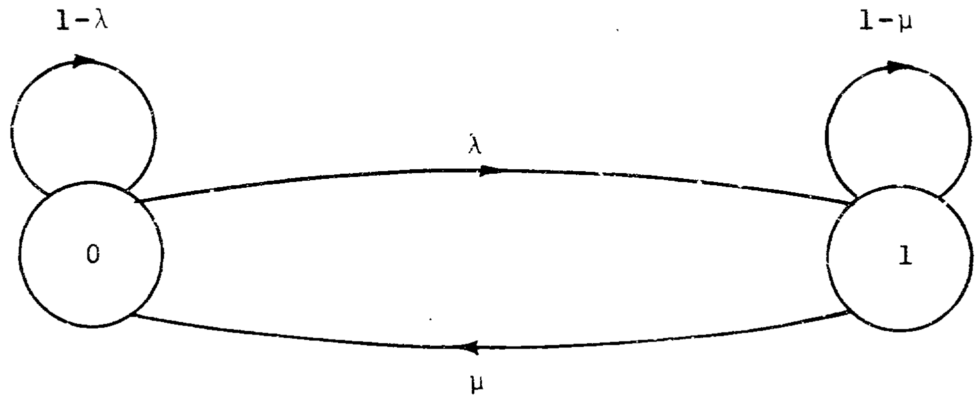

Fig. 4.1. Markov Graph for a System Consisting of One Unit

was in State 1 at time t and returned to State 0 in t,t+dt, thus we have:

$$P_0(t+dt) = P_0(t)(1-\lambda dt) + P_1(t)\mu dt+0(dt) \qquad (4.2)$$

Similarly, the probability of being in State 1 at time t+dt is derived from the probability that the system was in State 0 at time t and failed in t,t+dt; or it was in State 1 at time t, and the repair was not completed in t,t+dt. Therefore,

$$P_1(t+dt) = P_0(t)\lambda dt + P_1(t)(1-\mu dt) + 0(dt) \qquad (4.3)$$

The term 0(dt) in both equations represents the probability of two events taking place in t,t+dt, which is negligible so we can write the differential equations in the form:

$$\begin{aligned} P_0'(t) &= -\lambda P_0(t) + \mu P_1(t) \\ P_1'(t) &= \lambda P_0(t) - \mu P_1(t) \end{aligned} \qquad (4.4)$$

where:

$P_i(t)$ is the probability of being in State i at time t, and

$P_i'(t)$ is the first-order derivative with respect to t.

Shooman [165] has described a simple algorithm for writing the above equations and it is to equate the derivative of the probability at any node to the sum of the transitions coming into the node. Any unity gain factor of the self loops must first be set to zero and the dt factors are dropped from the branch gains.

Let the system be in State 0 (in operation) at time t, then the initial conditions are: $P_0(0) = 1$, $P_1(0) = 0$. Transforming Equations (4.4) into Laplace transforms under the initial conditions we have,

$$\begin{aligned} sP_0(s) - 1 + \lambda P_0(s) - \mu P_1(s) &= 0 \\ sP_1(s) \qquad - \lambda P_0(s) + \mu P_1(s) &= 0 \end{aligned} \tag{4.5}$$

and simplifying

$$\begin{aligned} (s+\lambda)P_0(s) - \mu P_1(s) &= 1 \\ -\lambda P_0(s) + (s+\mu)P_1(s) &= 0 \end{aligned} \tag{4.5}$$

Using Cramer's rule,

$$P_0(s) = \frac{\begin{vmatrix} 1 & -\mu \\ 0 & s+\mu \end{vmatrix}}{\begin{vmatrix} s+\lambda & -\mu \\ -\lambda & s+\mu \end{vmatrix}}$$

and

$$P_0(s) = \frac{s+\mu}{s(s+\lambda+\mu)} \tag{4.7}$$

Now the availability function A(t) will be the inverse transform of $P_0(s)$:

$$A(t) = P_0(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \tag{4.8}$$

In many cases we are interested in the average uptime for some definite period of time. This can be found simply by summing A(t) over the time interval of interest and dividing by the total time.

$$A(T) = \frac{1}{T}\int_0^T A(t)\,dt$$

In this instance, we have:

$$A(T) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 T} - \frac{\lambda}{(\lambda+\mu)^2}\, e^{-(\lambda+\mu)T} \qquad (4.9)$$

If we are interested in the long-term availability of the system we can let $T \to \infty$ and find

$$A(\infty) = \frac{\mu}{\lambda+\mu} \qquad (4.10)$$

Systems Subject to Two Types of Repair

Consider the problem where an equipment is subject to two types of repair. When the equipment fails for the first time a partial repair is performed which restores the system to operation; however, it increases the probability of failure. After it fails the second time, a complete repair is performed which restores the equipment to a "good-as-new" condition. Let λ_1 be the failure rate when the equipment has been through a complete repair, and λ_2 when it has been through a partial repair ($\lambda_2 > \lambda_1$).

Similarly, let μ_1 be the repair rate for a partial repair, and μ_2 be the repair rate for a complete repair ($\mu_2 < \mu_1$). To formulate the problem we establish four states in which the system can be at any time: (1) State 0--the system is operating after a complete repair has been performed; (2) State 1--the system is failed and partial repair is being performed, (3) State 2--the system is operating after the completion of a partial repair, and (4) State 3--the system is failed and a complete repair is being performed. Figure 4.2 depicts the system states. It has to be noticed that State 0 and State 2 constitute acceptable system states.

The transition matrix is:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1-\lambda_1 & \lambda_1 & 0 & 0 \\ 0 & 1-\mu_1 & \mu_1 & 0 \\ 0 & 0 & 1-\lambda_2 & \lambda_2 \\ \mu_2 & 0 & 0 & 1-\mu_2 \end{pmatrix} \end{array} \quad (4.11)$$

The resulting system of differential equations is

$$\begin{aligned} P_0'(t) &= -\lambda_1 P_0(t) + \mu_2 P_3(t) \\ P_1'(t) &= \lambda_1 P_0(t) - \mu_1 P_1(t) \\ P_2'(t) &= \mu_1 P_1(t) - \lambda_2 P_2(t) \\ P_3'(t) &= \lambda_2 P_2(t) - \mu_2 P_3(t) \end{aligned}$$

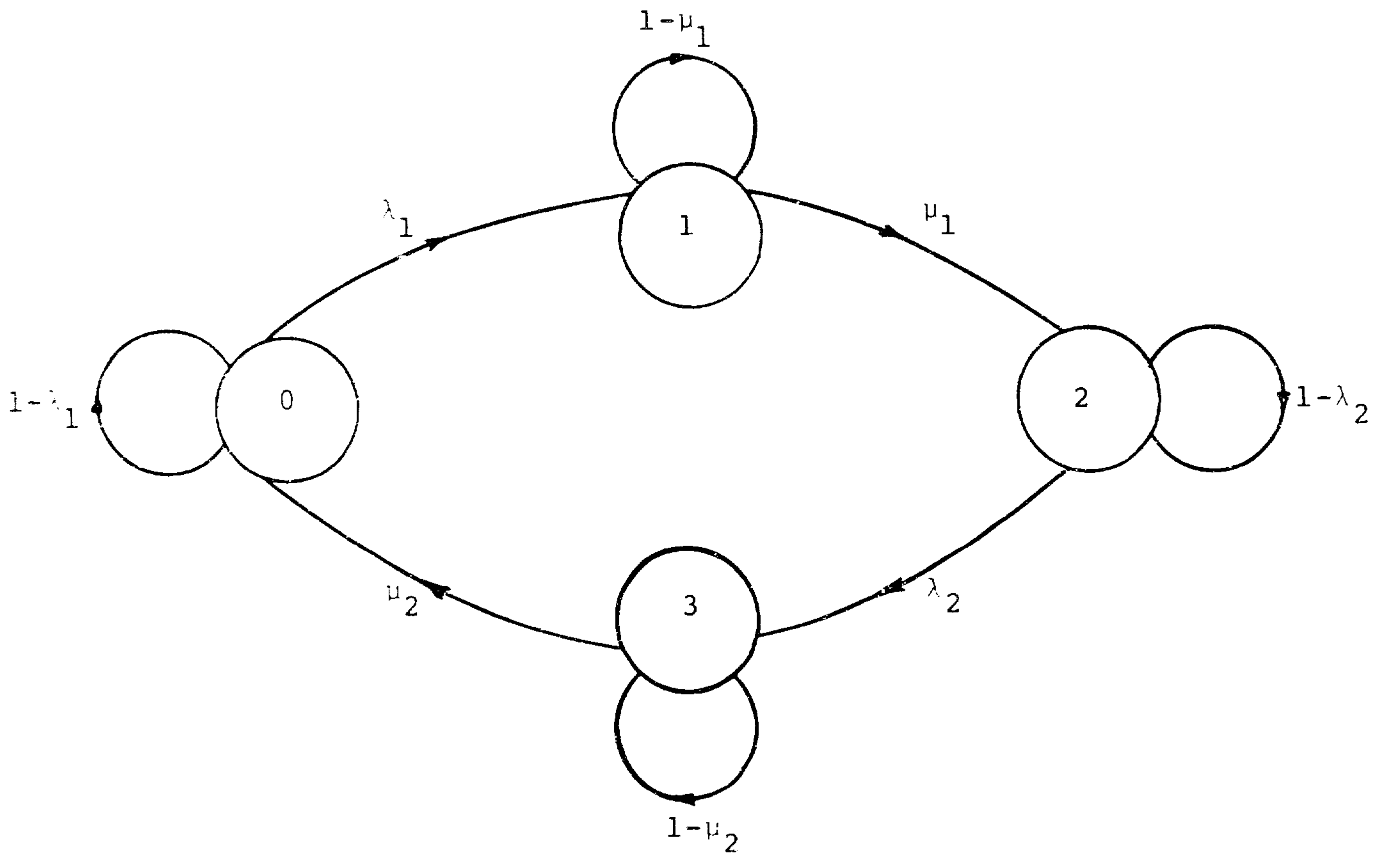

Fig. 4.2. Markov Graph for a System with Two Different Failure and Repair Times

For steady state behavior it can easily be shown that the limit of $P_i(t)$ always exists; i.e., $P_i = \lim_{t\to\infty} P_i(t)$. This means that the steady state solutions can be found by setting the derivatives $P_i'(t)$ equal to zero. Then the system of differential equations reduces to a system of algebraic equations. So Equations (4.12) can be reduced to the following system of algebraic equations:

$$
\begin{aligned}
0 &= -\lambda_1 P_0 + \mu_2 P_3 \\
0 &= \lambda_1 P_0 - \mu_1 P_1 \\
0 &= \mu_1 P_1 - \lambda_2 P_2 \\
0 &= \lambda_2 P_2 - \mu_2 P_3
\end{aligned}
\tag{4.13}
$$

To solve these equations we must also make use of the fact that the P_i's are a probability distribution; i.e.,

$\sum_{i=0}^{n} P_i = 1$. So adding this equation to the above system of algebraic equations and solving, we can find the steady-state availability

$$A(\infty) = P_0 + P_2$$

$$A(\infty) = \frac{2\lambda_1\mu_1\mu_2}{\lambda_1\lambda_2\mu_1+\lambda_2\mu_1\mu_2+\lambda_1\lambda_2\mu_2+\lambda_2\mu_1\mu_2} \tag{4.14}$$

It can be seen that if $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$ of Equation (4.14) reduces to $\mu/\lambda+\mu$, which is the same value in the previous model.

System with Series Configurations

Consider the simple system where two equipments are connected in series such that if either fails the system fails. For simplicity, we shall assume that each equipment fails at the same rate, λ, and can be repaired at the same rate, μ. Now the system can be thought of as being in any one of three possible states at some time, t: (1) State 0--when both equipments are operating; (2) State 1--when one equipment is operating and the second is under repair; and (3) State 2--when both equipments are under repair.

Since both equipments are required, the system is defined as down when it reaches State 1. Thus, $A(t) = P_0(t)$, the probability that the system is in State 0 at time, t.

The availability function is directly influenced by the number of repairmen available to service the failed equipments. So we will consider first the case when there is a single repairman, and then when there are two repairmen working independently or working together.

One Repairman Case

When a single repairman is available to service the two equipments, the system transition matrix P is:

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & \mu & 1-\mu \end{pmatrix} \end{array} \qquad (4.15)$$

The system can be depicted as in Figure 4.3. The resulting system of differential equations is:

$$
\begin{aligned}
P_0'(t) &= -2\lambda P_0(t) + \mu P_1(t) \\
P_1'(t) &= 2\lambda P_0(t) - (\lambda+\mu)P_1(t) + \mu P_2(t) \\
P_2'(t) &= \lambda P_1(t) - \mu P_2(t)
\end{aligned}
\tag{4.16}
$$

As mentioned before, this system of differential equations can be solved using Laplace transforms. In order to obtain the steady-state availability, the steady-state solutions can be found by letting the derivatives equal zero and using the fact that the system must be in one of the mutually exclusive states $P_0 + P_1 + P_2 = 1$. Therefore, the system will be reduced to the following system of algebraic equations:

$$
\begin{aligned}
0 &= -2\lambda P_0 + \mu P_1 \\
0 &= 2\lambda P_0 - (\lambda+\mu)P_1 + \mu P_2 \\
0 &= \lambda P_1 - \mu P_2 \\
1 &= P_0 + P_1 + P_2
\end{aligned}
\tag{4.17}
$$

Solving for P_0, P_1 and P_2 we have,

$$P_0 = \frac{\mu^2}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

$$P_1 = \frac{2\lambda\mu}{\mu^2 + 2\lambda\mu + 2\lambda^2}$$

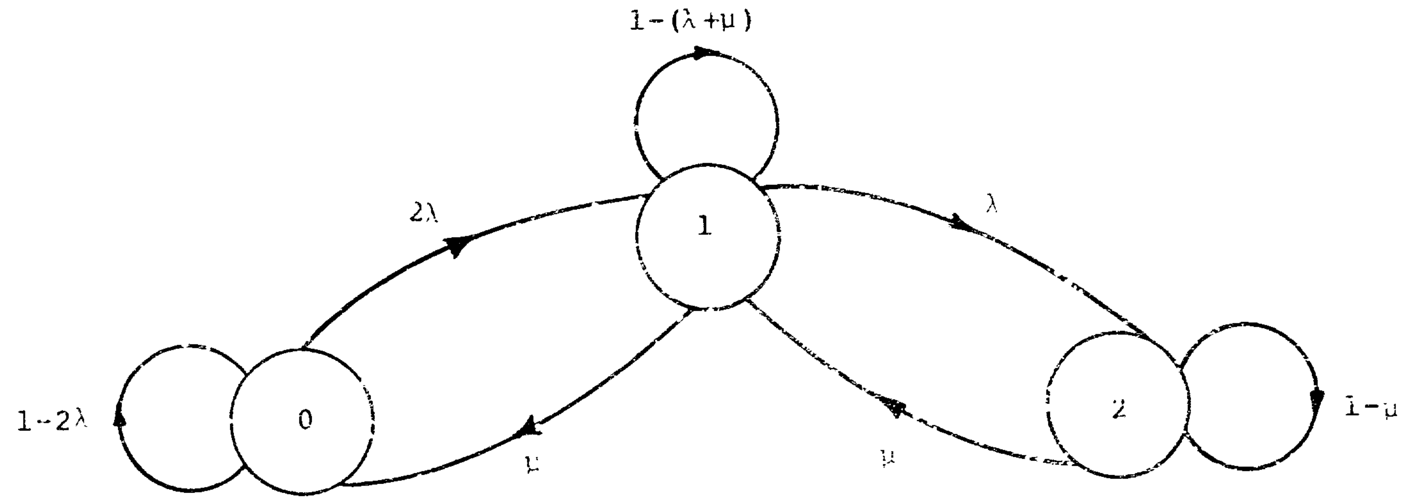

Fig. 4.3. Markov Graph for a System with Two Components in Series and One Repairman

$$P_2 = \frac{2\lambda^2}{\mu^2 + 2\lambda\mu + 2\lambda^2} \tag{4.18}$$

The steady-state availability, $A(\infty)$, will be:

$$A(\infty) = P_0 = \frac{\mu^2}{\mu^2 + 2\mu\lambda + 2\lambda^2} \tag{4.19}$$

Next we will consider the case of two equipments in series with two repairmen.

Two Equipments in Series With Two Repairmen

First, we will consider the case where each repairman can only work on one particular equipment. The Markov graph of this system is depicted in Figure 4.4. The transition matrix P of this system is:

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 2\mu & 1-2\mu \end{pmatrix} \end{array} \tag{4.20}$$

The difference between Equations (4.15) and (4.20) is in the last row. This occurs because if we are in State 2 at time, t, we can return to State 1 if either of the equipments is repaired.

The steady-state equations of this system are:

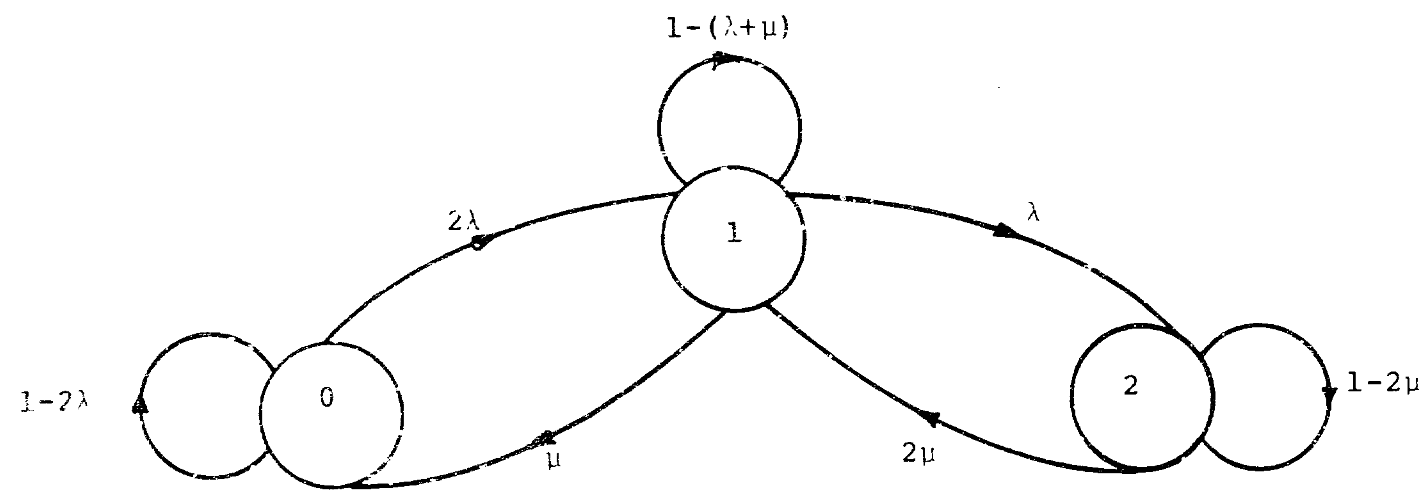

Fig. 4.4. Markov Graph for a System with Two Components in Series and Two Repairmen

$$
\begin{aligned}
0 &= -2\lambda P_0 + \mu P_1 \\
0 &= 2\lambda P_0 - (\lambda+\mu)P_1 + 2\mu P_2 \\
0 &= \lambda P_1 - 2\mu P_2 \\
1 &= P_0 + P_1 + P_2
\end{aligned}
\tag{4.21}
$$

Solving, we find that:

$$A(\infty) = P_0 = \frac{\mu^2}{(\lambda+\mu)^2} \tag{4.22}$$

Joint Servicing of Failed Equipments

In the previous case if the two repairmen do not work independently of each other, i.e., if there are two equipment series systems with two repairmen, we might expect that both of them would attempt to service the equipment that failed. The only time they would work independently is when both equipments have failed. Sandler [157] assumed that if two repairmen are servicing a single equipment, the repair rate is 1.5μ. Under the assumption that if both repairmen are servicing a single equipment and a second one fails, the second repairman immediately returns to service his own equipment. In this case, the transition matrix will be:

$$
P = \begin{array}{c c}
 & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \end{array} &
\begin{pmatrix}
1-2\lambda & 2\lambda & 0 \\
1.5\mu & 1-(1.5\mu+\lambda) & \lambda \\
0 & 2\mu & 1-2\mu
\end{pmatrix}
\end{array}
\tag{4.23}
$$

The steady state equations of this system are:

$$
\begin{aligned}
0 &= -2\lambda P_0 + 1.5\mu P_1 \\
0 &= 2\lambda P_0 - (1.5\mu+\lambda) P_1 + 2\mu P_2 \\
0 &= \lambda P_1 - 2\mu P_2 \\
1 &= P_0 + P_1 + P_2
\end{aligned}
\qquad (4.24)
$$

Solving, we find that:

$$
A(\infty) = P_0 = \frac{3\mu}{3\mu + 4\lambda\mu + 2\lambda^2} \qquad (4.25)
$$

Availability Models of Parallel Redundant Configurations

Consider a two-equipment redundant system operating in parallel which can be in the following states: (1) State 0--both equipments operating, (2) State 1--one equipment operating and one equipment under repair, and (3) State 2--both equipments under repair.

When the system is in State 2 it is defined as failed. The transition diagram is depicted in Figure 4.5. The transition matrix is developed in the same manner as before. The transition matrix P is:

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}
\begin{array}{c}
\begin{array}{ccc} 0 & 1 & 2 \end{array} \\
\begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 2\mu & 1-2\mu \end{pmatrix}
\end{array}
\qquad (4.26)
$$

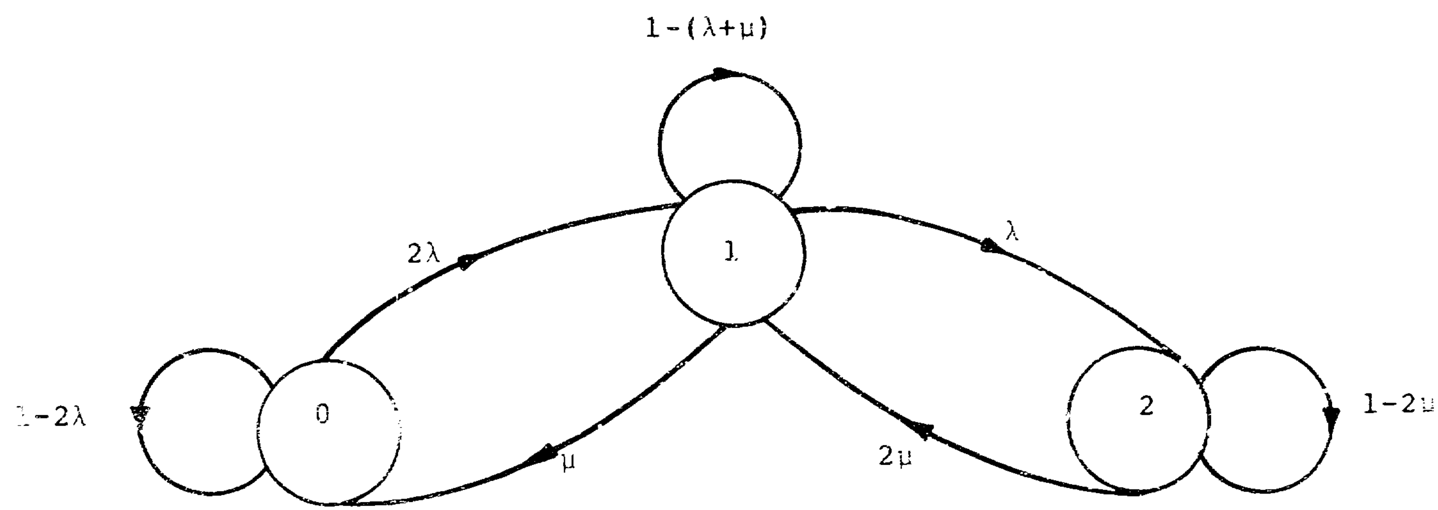

Fig. 4.5. Markov Graph for a System with Two Components in Parallel and Two Repairmen

The transition matrix leads directly to the system of linear homogenous differential equations which describe the stochastic behavior of this system and are as follows:

$$\begin{aligned}
P_0'(t) &= -2\lambda P_0(t) + \mu P_1(t) \\
P_1'(t) &= 2\lambda P_0(t) - (\lambda+\mu)P_1(t) + 2\mu P_2(t) \\
P_2'(t) &= \lambda P_1(t) - 2\mu P_2(t)
\end{aligned} \qquad (4.27)$$

Considering the initial condition, let the system be in State 0 at time 0, then

$$P_0(0) = 1, \quad P_1(0) = 0, \quad P_2(0) = 0$$

Taking Laplace transforms of Equations (4.27),

$$\begin{aligned}
sP_0(s) - P_0(0) &= -2\lambda P_0(s) + \mu P_1(s) \\
sP_1(s) - P_1(0) &= 2\lambda P_0(s) - (\lambda+\mu)P_1(s) + 2\mu P_2(s) \\
sP_2(s) - P_2(0) &= \lambda P_1(s) - 2\mu P_2(s)
\end{aligned} \qquad (4.28)$$

Using the initial conditions, we obtain:

$$\begin{aligned}
(s+2\lambda)P_0(s) - \mu P_1(s) &= 1 \\
-2\lambda P_0(s) + (s+\lambda+\mu)P_1(s) - 2\mu P_2(s) &= 0 \\
-\lambda P_1(s) + (s+2\mu)P_2(s) &= 0
\end{aligned} \qquad (4.29)$$

Solving, using Cramer's rule, we obtain:

$$P_2(s) = \frac{\begin{vmatrix} s+2\lambda & -\mu & 0 \\ -2\lambda & s+\lambda+\mu & 0 \\ 0 & -\lambda & 0 \end{vmatrix}}{\begin{vmatrix} s+2\lambda & -\mu & 0 \\ -2\lambda & s+\lambda+\mu & -2\mu \\ 0 & -\lambda & s+2\mu \end{vmatrix}} \tag{4.30}$$

Thus,

$$P_2(s) = \frac{2\lambda^2}{s(s+2\lambda + 2\mu)(s+\lambda+\mu)} \tag{4.31}$$

Breaking this expression into partial fractions we obtain:

$$\frac{2\lambda^2}{s(s+2\lambda+2\mu)(s+\lambda+\mu)} = \frac{A}{s} + \frac{B}{s+2\lambda+2\mu} + \frac{C}{s+\lambda+\mu} \tag{4.32}$$

(let $a = \lambda+\mu$)

$$= \frac{As^2 + 3asA + 2a^2A + Bs^2 + Bsa + Cs^2 + 2asC}{s(s+2a)(s+a)}$$

Equating constant terms we have

$$A = \frac{\lambda^2}{(\lambda+\mu)^2} \tag{4.33}$$

Equating coefficients of s and s^2 we obtain

$$B = \frac{\lambda^2}{(\lambda+\mu)^2} \tag{4.34}$$

$$C = \frac{2\lambda^2}{(\lambda+\mu)^2} \tag{4.35}$$

Hence,

$$P_2(s) = \frac{\lambda^2}{(\lambda+\mu)^2} \cdot \frac{1}{s} + \frac{\lambda^2}{(\lambda+\mu)^2} \cdot \frac{1}{(s+2\lambda+2\mu)} - \frac{-2\lambda^2}{(\lambda+\mu)^2} \cdot \frac{1}{(s+\lambda+\mu)} \tag{4.36}$$

Taking inverse Laplace transforms,

$$P_2(t) = \frac{\lambda^2}{(\lambda+\mu)^2} + \frac{\lambda^2}{(\lambda+\mu)^2} e^{-2(\lambda+\mu)t} - \frac{2\lambda^2}{(\lambda+\mu)^2} e^{-(\lambda+\mu)t} \tag{4.37}$$

Since $P_2(t)$ is the probability of being in the failed state at time t, the availability at time, t, is given by:

$$A(t) = 1 - P_2(t) = P_0(t) + P_1(t) \tag{4.38}$$

$$A(t) = \frac{\mu^2+2\lambda\mu}{(\lambda+\mu)^2} - \frac{\lambda^2 e^{-2(\lambda+\mu)t}}{(\lambda+\mu)^2} + \frac{2\lambda^2 e^{-(\lambda+\mu)t}}{(\lambda+\mu)^2} \tag{4.39}$$

From Equation (4.39) we obtain the steady-state expression:

$$A(\infty) = \lim_{T\to\infty} \int_0^T A(t)\,dt = \frac{\mu^2+2\lambda\mu}{(\lambda+\mu)^2} \tag{4.40}$$

In the two-equipment parallel system with two repairmen, we might expect both of them to work together if one unit failed. However, they would work independently if both units are failed. Thus, we may have the case that if a single repairman services a failed unit, the repair rate is μ, but if two repairmen service the same failed equipment the repair rate is 1.5μ [Sandler 157]. If we further assume that when both repairmen are servicing a single unit and the second one fails, the second repairman immediately returns to service his own unit, then the transition matrix is as follows:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ 1.5\mu & 1-(1.5\mu+\lambda) & \lambda \\ 0 & 2\mu & 1-2\mu \end{pmatrix} \end{array} \quad (4.41)$$

In this case it is assumed that failure of any unit was detected the instant it occurred. Very often this is not the case and the repair operation starts only when the entire system has failed.

Let us consider the model in which only one unit is repaired if the system of two units is parallel fails due to failure of both units. It is only when preventive maintenance is undertaken that the system is restored to the state where both units are operating. There is only one repairman. The Markov graph is shown in Figure 4.6 and the transition matrix is

$$P= \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 1-2\lambda & 2\lambda & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & \mu & 1-\mu \end{pmatrix} \end{array} \quad (4.42)$$

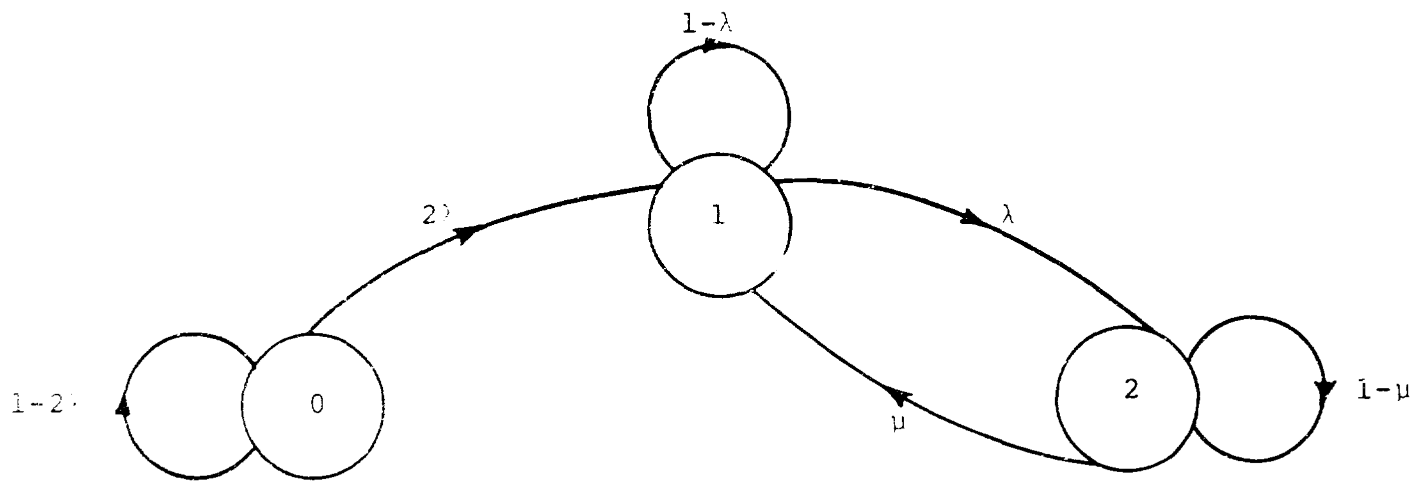

Fig. 4.6. Markov Graph for a System with Two Identical Units in Parallel and One Repairman, when only at System Failure, One Unit is Repaired

The differential equations are:

$$
\begin{aligned}
P_0'(t) &= -2\lambda P_0(t) \\
P_1'(t) &= 2\lambda P_0(t) - \lambda P_1(t) + \mu P_2(t) \\
P_2'(t) &= \lambda P_1(t) - \mu P_2(t)
\end{aligned}
\tag{4.43}
$$

Taking Laplace transforms and using the initial conditions $P_0(0) = 1$, $P_1(0) = 0$, and $P_2(0) = 1$, then:

$$
\begin{aligned}
(s+2\lambda)P_0(s) &= 1 \\
-2\lambda P_0(s) + (s+\lambda)P_1(s) - \mu P_2(s) &= 0 \\
-\lambda P_1(s) + (s+\mu)P_2(s) &= 0
\end{aligned}
\tag{4.44}
$$

and

$$
P_2(s) = \frac{\begin{vmatrix} s+2\lambda & 0 & 1 \\ -2\lambda & s+\lambda & 0 \\ 0 & -\lambda & 0 \end{vmatrix}}{\begin{vmatrix} s+2\lambda & 0 & 0 \\ -2\lambda & s+\lambda & -\mu \\ 0 & -\lambda & s+\mu \end{vmatrix}}
\tag{4.45}
$$

or

$$
P_2(s) = \frac{2\lambda^2}{s(s+2\lambda)(s+\lambda+\mu)}
$$

$$
= \frac{\lambda}{\lambda+\mu} \cdot \frac{1}{s} - \frac{\lambda}{(\mu-\lambda)} \cdot \frac{1}{(s+2\lambda)} + \frac{2\lambda^2}{(\mu^2-\lambda^2)} \cdot \frac{1}{(s+\lambda+\mu)}
\tag{4.46}
$$

Taking inverse Laplace transforms, we obtain:

$$P_2(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\mu-\lambda} e^{-2\lambda t} + \frac{2\lambda^2}{\mu^2-\lambda^2} e^{-(\lambda+\mu)t} \tag{4.47}$$

and

$$A(t) = 1 - P_2(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\mu-\lambda} e^{-2\lambda t} - \frac{2\lambda^2}{\mu^2-\lambda^2} e^{-(\lambda+\mu)t} \tag{4.48}$$

Now if in the system with two units in parallel and two repairmen, the status of the individual units is not monitored, repair will not begin until the system is in State 2 where both units have failed. We can define the four states with reference to the Markov graph shown in Figure 4.7 as follows: (1) State 0--both units are operating; (2) State 1--one unit is operating, one failed and has not been detected; (3) State 2--both units failed and are under repair; and (4) State 3--one unit is operating, one has failed and is under repair.

The transition matrix is:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 1-2\lambda & 2\lambda & 0 & 0 \\ 0 & 1-\lambda & \lambda & 0 \\ 0 & 0 & 1-2\mu & 2\mu \\ \mu & 0 & \lambda & 1-(\mu+\lambda) \end{pmatrix} \end{array} \tag{4.49}$$

The system of the differential equations is:

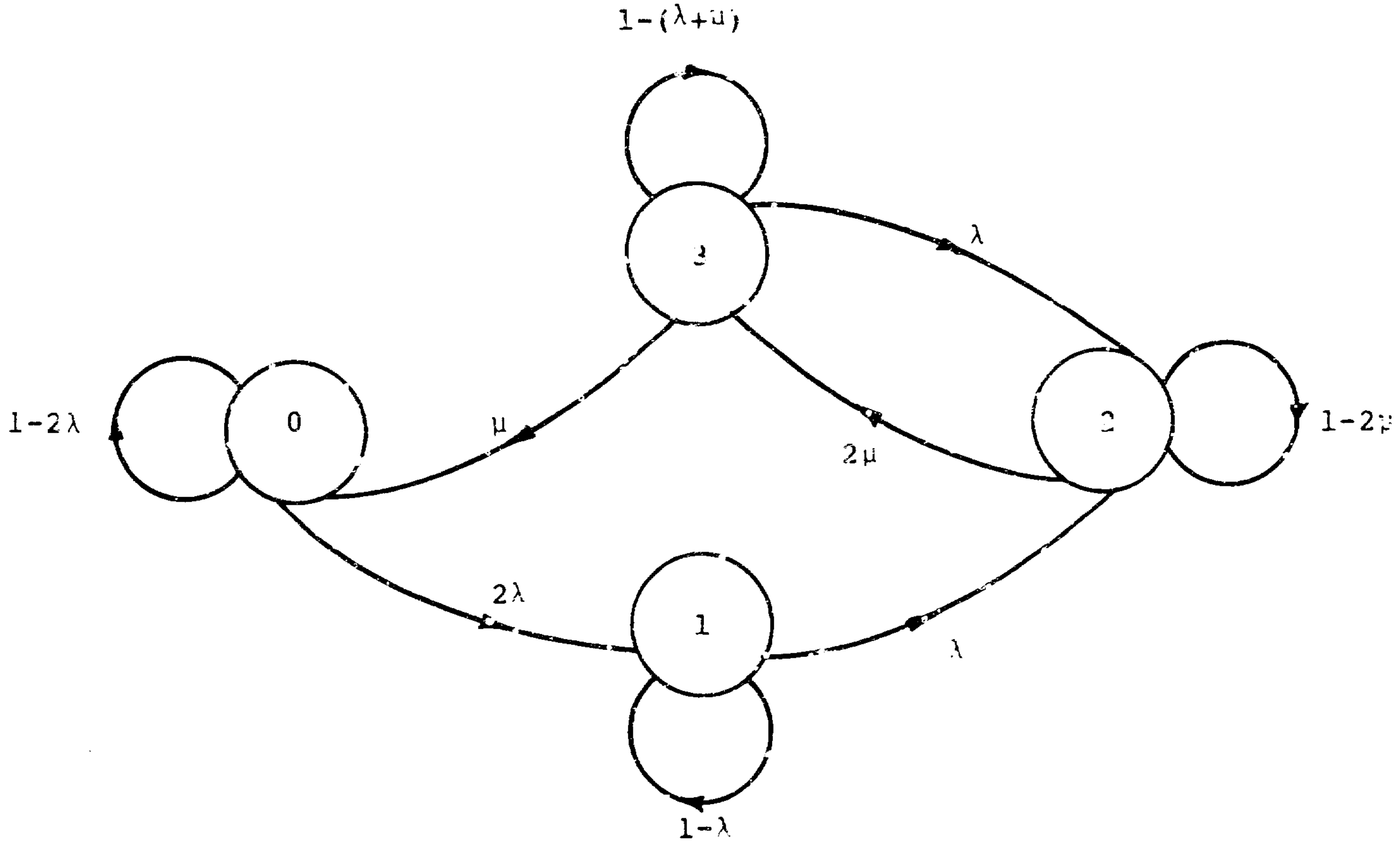

Fig. 4.7. Markov Graph for a System with Two Identical Units in Parallel and Two Repairmen, when only at System Failure, Both Units are Repaired

$$
\begin{aligned}
P_0'(t) &= 2\lambda P_0(t) && && + \mu P_3(t) \\
P_1'(t) &= 2\lambda P_0(t) - \lambda P_1(t) && && \\
P_2'(t) &= && \lambda P_1(t) - 2\mu P_2(t) && + \lambda P_3(t) \\
P_3'(t) &= && && 2\mu P_2(t) - (\mu+\lambda) P_3(t)
\end{aligned}
\tag{4.50}
$$

Taking the inverse Laplace transforms with the initial condition $P_0(0) = 1$, $P_1(0) = 0$, $P_2(0) = 0$, and $P_3(0) = 0$, we have

$$
\begin{aligned}
(s+2\lambda)P_0(s) & & & -\mu P_3(s) &= 1 \\
-2\lambda\ P_0(s) + (s+\lambda)P_1(s) & & & &= 0 \\
- & \lambda P_1(s) - (s+2\mu)P_2(s) & - & \lambda P_3(s) &= 0 \\
 & - & 2\mu P_2(s) + & (s+\mu+\lambda)P_3(s) &= 0
\end{aligned}
\tag{4.51}
$$

and

$$
P_2(s) = \frac{\begin{vmatrix} s+2\lambda & 0 & 1 & -\mu \\ -2\lambda & s+\lambda & 0 & 0 \\ 0 & -\lambda & 0 & -\lambda \\ 0 & 0 & 0 & s+\mu+\lambda \end{vmatrix}}{\begin{vmatrix} s+2\lambda & 0 & 0 & -\mu \\ -2\lambda & s+\lambda & 0 & 0 \\ 0 & -\lambda & s+2\mu & -\lambda \\ 0 & 0 & -2\mu & s+\mu+\lambda \end{vmatrix}}
\tag{4.52}
$$

where the numerator $= 2\lambda^2(s+\mu+\lambda)$ and

the denominator $= s(s+3\lambda)\{s^2+s(3\mu+\lambda)+2\mu^2\}$.

The solution for the roots of $s^2 + s(3\mu+\lambda) + 2\mu^2$ yields

$$r_1, r_2 = \frac{-(3\mu+\lambda) \pm \sqrt{(3\mu+\lambda)^2 - 8\mu^2}}{2} \qquad (4.53)$$

Hence,

$$P_2(s) = \frac{2\lambda^2(s+\mu+\lambda)}{s(s+3\lambda)(s-r_1)(s-r_2)} \qquad (4.54)$$

Breaking this expression into partial fractions,

$$P_2(s) = \frac{A}{s} + \frac{B}{s+3\lambda} + \frac{C}{s-r_1} + \frac{D}{s-r_2} \qquad (4.55)$$

The values of A, B, C, and D can be found by supression. Taking the inverse Laplace transforms, we obtain,

$$P_2(t) = A + B\,e^{-3\lambda t} + C\,e^{r_1 t} + D\,e^{r_2 t} \qquad (4.56)$$

and the availability is given by:

$$A(t) = 1 - P_2(t) \qquad (4.57)$$

Inspection of the quadratic equation for r_1, r_2 shows that r_1 and r_2 are always negative real numbers since λ and μ are always positive; therefore, all the time horizons are decaying exponentially and the instantaneous availability, A(t), rapidly converges to the steady-state value.

Equation (4.56) is complex in nature due to r_1 and r_2 not having simple forms and, consequently, it is not easy to obtain the steady-state availability from Equation (4.57). But the steady-state availability may be obtained by studying the steady-state behavior. This steady-state solution can be found by setting the derivatives $P_i'(t)$ equal to zero. Then the system of differential equations reduces to a system of algebraic equations. The additional fact that P_i's are a probability and hence

$\sum_{i=0}^{n} P_i = 1$ needs to be used where n is the number of possible states. So to obtain the steady-state availability the set of equations is:

$$
\begin{aligned}
0 &= -2\lambda P_0 && && + \mu P_3 \\
0 &= 2\lambda P_0 - \lambda P_1 && && \\
0 &= && \lambda P_1 - 2\mu P_2 && + \lambda P_3 \\
0 &= && 2\mu P_2 && - (\lambda+\mu) P_3 \\
1 &= P_0 + P_1 + && P_2 && + P_3
\end{aligned}
\tag{4.58}
$$

Solving for P_2 using the last four equations,

$$
P_2 = \frac{\begin{vmatrix} 2\lambda & -\lambda & 0 & 0 \\ 0 & \lambda & 0 & \lambda \\ 0 & 0 & 0 & -(\lambda+\mu) \\ 1 & 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2\lambda & -\lambda & 0 & 0 \\ 0 & \lambda & -2\mu & \lambda \\ 0 & 0 & 2\mu & -(\lambda+\mu) \\ 1 & 1 & 1 & 1 \end{vmatrix}} = \frac{2\lambda(\lambda^2+\lambda\mu)}{6\mu\lambda^2+2\lambda^3+6\lambda\mu^2}
$$

$$P_2 = \frac{\lambda^2+\lambda\mu}{\lambda^2+3\mu\lambda+3\mu^2} \tag{4.59}$$

The steady-state availability is:

$$A(\infty) = 1 - P_2 = \frac{2\mu\lambda+3\mu^2}{\lambda^2+3\mu\lambda+3\mu^2} \tag{4.60}$$

Many complex problems can similarly be solved for the steady-state availability without too much difficulty.

Availability of Standby Redundant Configurations

Standby redundancy assumes that the off-line equipment(s) either cannot fail or have a failure rate less than on-line equipments. When this is true, we would expect a system's availability to be greater with standby redundancy than with parallel redundancy. Consider a two-equipment standby system where the on-line equipment fails at the rate, λ, and the off-line equipment cannot fail until it is switched to an on-line position. Assuming perfect switch reliability, the transition diagram for this system is depicted in Figure 4.8.

The transition matrix for this system is:

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & \quad 1 & \quad 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1-\lambda & \lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & \mu & 1-\mu \end{pmatrix} \end{array} \tag{4.61}$$

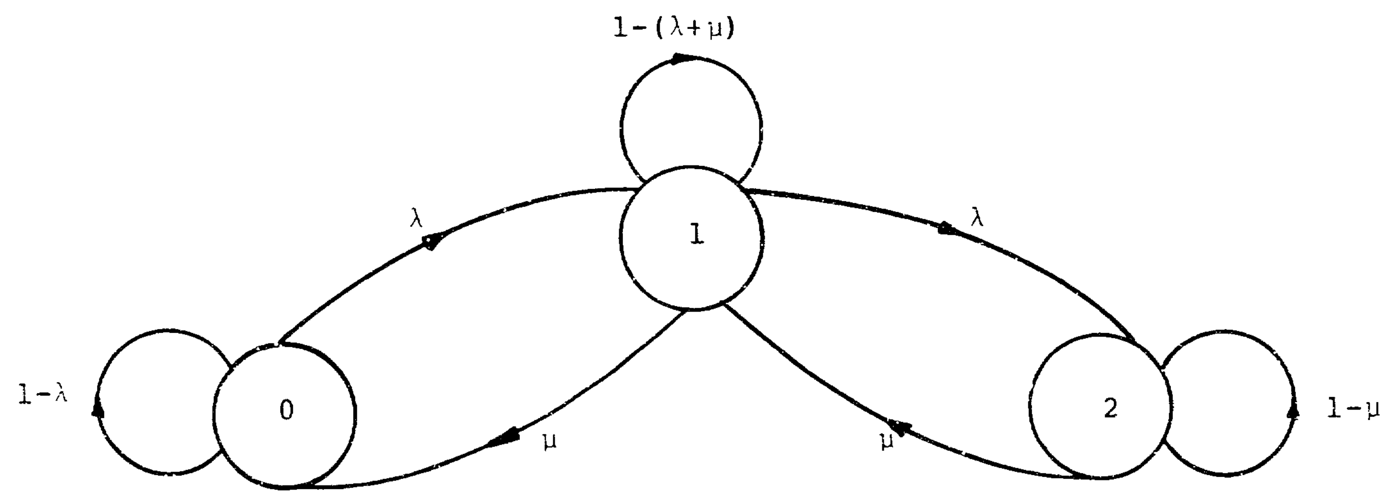

Fig. 4.8. Markov Graph for a System with Two Components in Standby Configuration with One Repairman

The steady-state equation of this system is:

$$
\begin{aligned}
0 &= -\lambda P_0 + \mu P_1 \\
0 &= \lambda P_0 + 1 - (\lambda+\mu) P_1 + \mu P_2 \\
0 &= + \lambda P_1 + \mu P_2 \\
1 &= P_0 + P_1 + P_2
\end{aligned}
\tag{4.62}
$$

The steady-state availability can be found as:

$$A(\infty) = P_0 + P_1 = 1 - P_2$$

$$A(\infty) = \frac{\mu^2}{\mu^2 + \lambda\mu + \lambda^2} \tag{4.63}$$

CHAPTER V

CORRECTIVE AND PREVENTIVE MAINTENANCE AND OPTIMIZATION TECHNIQUES

Effect of Corrective and Preventive Maintenance

At one time or another all recoverable systems are subject to some form of maintenance. In general, there are two categories of maintenance actions. The first is off-shedule or corrective maintenance and is performed whenever there is an inservice failure or malfunction. The system operation is restored by replacing, repairing or adjusting the component or components which caused the interruption of service. The second category is the scheduled or preventive maintenance and is performed at regular intervals to keep the system in a condition consistent with its built-in levels of performance reliability and safety. According to Bazovsky [14], during preventive maintenance, servicing, and inspection, minor and major overhauls are done such that

1. regular care is provided to normally operating subsystems and components which require such attention (lubrication, refueling, cleaning, adjustment, alignment, etc.);
2. failed redundant components are checked, replaced, or repaired if the system contains redundancy; and
3. components which are nearing a wearout condition are replaced or overhauled.

Preventive maintenance is usually associated with wearout failures. Preventive maintenance policies consist of some action depending upon either the operating age of certain components in the system, the state of the system degradation, or the system configuration. In the first case, a preventive maintenance policy is usually some program for the planned replacement or repair of certain critical components after they have accumulated a given number of operating hours. In the second case, the preventive maintenance policies are designed to minimize the time the system will spend in the degraded state. In the third case, the preventive maintenance policies consist of periodic inspection and repair to increase the mean life of the system.

Planned replacements or maintenance actions are advantageous for systems and parts whose failure rate increase with time, or are less costly to replace or repair when operating than after failure. Under preventive maintenance policies it may be possible either to increase a piece of equipment's availability or reliability or to minimize the total cost of replacement and repairs. Thus, one of the most important maintenance problems is that of specifying a maintenance policy which balances the cost of failures against the cost of preventive maintenance actions in order to minimize total maintenance cost. For preventive maintenance to be worthwhile, the failure rate of the system must increase over time or the preventive maintenance of the system must cost less than the corrective maintenance. Normally, preventive maintenance for a component is assumed to have the same effect as the replacement of the component. In general, four different types of preventive maintenance are possible (see Table 5.1).

TABLE 5.1

TYPE OF PREVENTIVE MAINTENANCE

Type of Preventive Maintenance	References
Block replacement type	10, 17, 39, 175, 185, 195
Age replacement type	8, 11, 1[illegible] 38, 41, 52, 112, 125, 133, 155, 181-184, 195
Random periodic replacement type	10, 26, 64, 78, 182, 183 195
Sequentially determined replacement type	8, 10-12, 98, 195

In block replacement, all components of a given type are replaced (or repaired) simultaneously at times independent of the failure history of the system. This policy is perhaps more realistic than others since it does not require the keeping of records on component use, but it has the undesirable characteristic that relatively new components are replaced. This method is sometimes called minimal repair-replacement type because for failure only a minimal repair is done, then the system is always replaced at age T. By definition, a minimal repair does not affect the hazard rate of the system but it enables the system to continue its work. It is often called "bad as old."

In age replacement, we replace a component exactly at the time of failure or at T hours after its installation (previous replacement or previous preventive maintenance), whichever occurs first (T is constant). The random periodic policy differs only in that T is a random variable. Gopalan and D'Souza [78] have found the availability and reliability of a 1-server 2-unit system subject to preventive maintenance and repair under the assumption that the pdf's of the times to failure and to preventive maintenance of a unit are arbitrary, while the repair and preventive maintenance rates are constant but different. Gopalan and Venkatachalam [81] extended this work to a n-unit system and also they analyzed a n-unit system in which each unit consists of two components connected in series. The sequentially determined replacement policy is one in which the replacement interval is determined at each removal (or preventive maintenance) in accord with the time remaining to the time span.

The earliest approach to the planned replacement problem was done by Campbell [36] and Welker [185]. It is concerned with mass replacement, and develops a method for determining optimum replacement intervals for certain vacuum tubes. Optimum block replacement policies for an infinite time span is also studied by Savage [161]. A theory of optimum sequential replacement policies for the case of a finite time horizon has been developed by Barlow and Proschan [12]. They show that for a finite time horizon there exists policies which require that after each removal the next planned replacement interval is selected to minimize the expected expenditure during the remaining time, and that these policies will be more effective than a fixed replacement policy. However, periodic or preventive maintenance policies assuming an infinite usage horizon seem to have received the most attention in the literature.

Earlier work on restricted forms of preventive maintenance problems is found in Reference 181. In a series of reports, Weiss [181-183] considers the effect on system reliability and on the maintenance costs of both strictly periodic and random periodic maintenance or replacement policies for an essentially infinite usage period. The operating characteristic of random periodic policies is determined by Flehinger [64]. Derman and Sacks [52] obtain the optimal replacement policy for a piece of equipment in which the decision to replace depends upon the observed state of the equipment deterioration at specified points in time. The derivation of an optimum periodic maintenance interval corresponding to a given finite span is basically a much more difficult problem. Barlow and Proschan [11] prove the existence of such an optimal policy. Further, they carefully expose the strictly periodic and random periodic maintenance problems, and have shown that for an infinite time horizon there always exists a strictly periodic maintenance policy which is superior to a random policy [12].

Meyers and Dick [120] have studied the effects of preventive maintenance on availability for a system composed of similar components where at least n out of m components must operate for the system to function. Nakagawa and Osaki [132] have dealt with optimal preventive maintenance policies to maximize the availability for a 2-unit redundant system.

Optimal Allocation of Availability Parameters

As the high degree of complexity is involved in many of the modern-day systems, much interest has been shown in allocating the availability parameters at component levels in the early stages of system design. The practical problem is to determine those parameters from a design, redesign or operating point of view so that some measure such as cost or weight of the system is minimized while a system availability requirement is met. Various combinations of availability parameters are used as decision variables in the allocation problem (see Table 5.2).

TABLE 5.2

AVAILABILITY PARAMETERS
(decision variables in the model)

Availability Parameters	References
MTBF and MTTR	96, 119, 145, 164, 190, 193
Numbers of redundant components	94, 138, 157
MTBF, MTTR, and number of redundant components	75, 110
Failure rate, repair rate, and preventive maintenance period	39, 175, 176
Failure rate, mean corrective maintenance time, mean preventive maintenance time, and age for preventive maintenance	112

The optimization techniques employed for the availability allocation problem are summarized in Table 5.3.

TABLE 5.3

OPTIMIZATION TECHNIQUE EMPLOYED FOR AVAILABILITY ALLOCATION

Optimization Technique	References
Dynamic Programming	94, 110, 157, 164, 190
Integer programming	160
Geometric programming	96
Lagrange multipliers	75, 119, 164, 176
SUMT	38, 39, 112, 175

The tradeoff technique between reliability and maintainability is discussed by Goldman and Whitin [75]. They employed Lagrange multipliers and show how the availability parameters consistent with the minimum cost operation and the specified system availability can be calculated. Kabak [96] has used geometric programming to determine the optimal design parameters that minimize total system cost.

Johnson [94] presents a methodology for optimizing the cost function under the predetermined availability level. McNichols and Messer, Jr. [119] have employed a cost-based procedure for allocating the availability parameters at components level. The allocation problem is expressed as the minimization of the total improvement cost, subject to the constraint of meeting the system availability goal, and is solved using the Lagrange multipliers method. This allocation technique is applicable to systems which can be described as a series model; that is, all components are necessary for proper system functioning. Extension to other models has not been considered although it appears feasible and would greatly expand the usefulness and application areas of the allocation problem.

It it also assumed that the individual components exhibit constant failure rates and that failures occur independently. The removal of these assumptions would generalize the allocation procedure and certainly make it more realistic. However, without the constant failure rate assumption, analytic solutions are usually not feasible and often impossible. The effects of various modes o ilure could be investigated by careful analysis and pred ion of possible failure patterns, and subsequent determination of the effect of these on the system availability.

The cost equations used in this development describe the costs associated with the improvement of component failure rates and repair times from achieved levels. Thus, the availability requirement is attained in the manner that requires the least cost in improvement of design and equipment. Although this problem is important to design and development groups, the allocations should be made on the basis of minimizing the cost of the system throughout its life. In this respect, the cost equations could be expanded to include the effects of component allocations on such costs of system ownership as sparing and downtime. The ultimate goal would be to allocate to the system components the levels of reliability and maintainability that would minimize the overall total system lifetime costs.

Shershin [164] has dealt with mathematical means for optimizing the simultaneous apportionments of reliability and maintainability by means of both Lagrange multipliers technique and dynamic programming.

Wilkinson and Valvekar [190] have also used dynamic programming for optimally allocating availability to a multicomponent system. They determine the MTBF and MTTR that minimize the system cost under the minimum availability requirement. As an extension of this study, Lambert et al. [110] present a method for determining the optimum MTBF, MTTR, and the number of redundant components for a multistage system to achieve a given availability at minimum cost by dynamic programming.

Tillman and Chatterjee [175] have studied the problem of allocating the failure rate, repair rate, and preventive maintenance period to each component of the system consisting of n subsystems in series where each subsystem has two identical components in parallel. An extension of this study can be seen in Reference 112, in which availability parameters consist of failure rate, mean corrective maintenance time, mean preventive maintenance time, and age for preventive maintenance of each component. Furthermore, a general series-parallel system configuration is considered. In both studies, the sequential unconstrained minimization technique (SUMT), which incorporates the Hooke and Jeeves pattern search and heuristic programming, employed

In Reference 38 not only the availability is considered, but both the availability and mean cycle-time are considered as constraints of the system. The objective is to maximize the system cost including the recurring and nonrecurring cost. In this study only the age replacement is considered, but the approach can be readily applied to other replacement policies. The problem is formulated and solved as a nonlinear programming problem.

Lie [112] studied the optimal availability allocation problem for a series-parallel system consisting of subsystems in series, where each subsystem has identical units in parallel having various probability density functions for failure and repair times of each unit. In developing the availability models, two types of maintenance policies for each subsystem are considered. The corrective maintenance is performed when the subsystem fails due to the failure of all redundant units and the preventive maintenance is scheduled at a fixed age of the subsystem and is actually performed only if the subsystem has not failed before this fixed age.

Preventive maintenance action consists of replacing or repairing only the failed units if each unit has a constant failure rate and replacing both failed and unfailed units if each unit has an increasing failure rate with time. Thus, each subsystem is assumed to be fully restored after the completion of either corrective or preventive maintenance. The cost of the system consists of three components--the cost for designing the mean time between maintenance and mean corrective and preventive maintenance time, the cost for corrective maintenance, and the cost for preventive maintenance. The optimal availability allocation problem, is then to determine individual units availability specifications which will minimize the total cost of the system under the constraint of meeting the system availability requirement. Both the cost function and the availability equation of the system are nonlinear; the optimization methods employed to solve this problem are both generalized reduced gradient (GRG) method and sequential unconstrained minimization technique (SUMT).

Summary and Recommendations

Summary

This thesis presents the results of an extensive literature review on availability of maintained systems. In Chapter II the different concepts and definition of availability is discussed; then a survey of the basic elements of availability is made to include the failure process, repair process and system configuration. The references are classified according to the last three elements. In Chapter III the different approaches used in obtaining availabilit methods are discussed. In

Chapter IV many availability models using the Markovian approach are presented. In Chapter V the effect of preventive maintenance policies on availability is explained and classification of the availability parameters used in the model and system optimization is presented.

Recommendations

While this survey covers a wide variety of topics on availability, there are some interesting areas for future research. One of the major areas is the situation when the Markovian conditions are not met or not approximately met and non-Markovian models must be used. Development in this area would permit the use of distributions other than the exponential. The whole area of non-perfect switching needs to be studied. The perfect switching models are the easiest to develop but, in practice, non-perfect switching cases are encountered.

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Availability appears to be a more appropriate measure than reliability for measuring the effectiveness of maintained systems because it includes reliability as well as maintainability. This thesis is a survey and a systematic classification of the literature relevant to

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) availability. Emphasis in this thesis is centered on a variety of topics related to availability. The topics discussed are: the definition and concepts of the availability, the probability density functions of failure times and of repair times, system configurations; and the various approaches employed to obtain the availability models; effect of preventive maintenance policies on availability; availability parameters in the model; and system optimization.